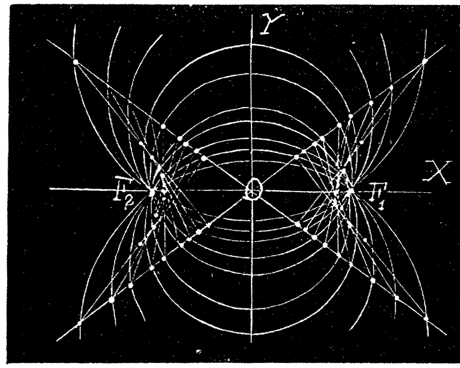
$$\frac{4b^2t_1^2}{(t_1^2-1)^2} \equiv b^2\left(\frac{t_1^2+1}{t_1^2-1}\right)^2 - b^2 \ldots . (11).$$

The lines joining the pairs of points (right hand, say) in which a system of co-axial circles, passing through the foci of an hyperbola, cuts the asymptotes, envelope that hyperbola.



Since, moreover, the middle point of the line joining (x_1, y_1), (x_2, y_2) lies on the hyperbola, we have the theorem:

The middle points of the lines joining the pairs of points in which a system of co-axial circles, passing through the foci of an hyperbola, cuts the asymptotes, describe that hyperbola.

These two theorems give two methods for constructing an hyperbola, the one by lines, the other by points, when the asymptotes and a focus are known.* Other constructions might readily have been given, but those given above seem the most instructive.

The University of Chicago, November, 1902.

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* Compare the November number of the MONTHLY for a note by the writer on the converse of this problem.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

163. Proposed by CHRISTIAN HORNUNG, A.M., Professor of Mathematics, Heidelberg University, Tiffin, O.

Three Dutchmen and their wives went to market to buy hogs. The names of the men were Hans, Klaus, and Hendricks, and of the women, Gertrude, Anna, and Katrine; but it was not known which was the wife of each man. They each bought as many hogs as each man or woman paid shillings for each hog, and each man spent three guineas more than his wife. Hendricks bought 23 hogs more than Gertrude, and Klaus bought 11 more than Katrine. What was the name of each man's wife?

Solution by J. SCHEFFER, A. M., Hagerstown, Md., and M. E. GRABER, Heidelberg University, Tiffin, O.

Let x represent the number of one of the women's hogs, and y the number of *her* husband's; then by the conditions of the problem $y^2=x^2+63$. Consequently x^2+63 must be an integer, since $\sqrt{(x^2+63)}$ represents the number of hogs. The equation $y^2-x^2=63$ or $(y+x)(y-x)=63$ admits of three solutions, viz., 63×1, 21×3, and 9×7.

$$\therefore\ \left.\begin{matrix} y+x=63 \\ y-x=1 \end{matrix}\right\} \text{ or } \left.\begin{matrix} y+x=21 \\ y-x=3 \end{matrix}\right\} \text{ or } \begin{matrix} y+x=9 \\ y-x=7; \end{matrix}$$

whence $y=32$, $x=31$; $y=12$, $x=9$; $y=8$, $x=1$. Since $32=9+23$ and $12=1+11$, 32 belongs to Hendricks, 12 to Klaus, 9 belongs to Katrine, 1 to Gertrude.

$\therefore$ 32 Hendricks, 31 Anna; 12 Klaus, 9 Katrine; 8 Hans, 1 Gertrude.

Hence, Hendricks is Anna's husband, Klaus is Katrine's husband, and Hans is Gertrude's husband.

Also solved by *W. R. LEBOLD*, *G. B. M. ZERR*, and *M. A. GRUBER*.

164. Proposed by JOSEPH V. COLLINS, Ph. D., Professor of Mathematics, State Normal School, Stevens Point, Wis.

Three women, the first with ten eggs, the second with thirty, and the third with fifty, went to market. They each got the same for their eggs, and all returned with the same money. What did they get?

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let a, b, and c=the respective numbers of eggs the three women sold at y cents for every d eggs; and, for the remaining eggs, let x cents=price per egg.

Then $(10-a)x+\dfrac{a}{d}y=(30-b)x+\dfrac{b}{d}y=(50-c)x+\dfrac{c}{d}y.$

Whence, $y=\dfrac{b-a-20}{b-a}dx=\dfrac{c-a-40}{c-a}dx=\dfrac{c-b-20}{c-b}dx.$

Solving for a, b, and c, we find $a+c=2b$. For *positive* values, $a<9$, $b>20+a$ and <31, $c>40+a$ and <51.

Put $d=2$. Take $a=2$; then $b=24$ and 26, $c=46$ and 50, $y=2x/11$ and $\frac{1}{3}x$. For integral values, put $x=11$ and 3, respectively; then $y=2$ and 1. Therefore, 10 eggs brought $\frac{2}{2}\times2c+8\times11c=90c$, or $\frac{2}{2}\times1c+8\times3c=25c$; 30 eggs brought $\frac{24}{2}\times2c+6\times11c=90c$, or $\frac{26}{2}\times1c+4\times3c=25c$; and 50 eggs brought $\frac{46}{2}\times2c+4\times11c=90c$, or $\frac{50}{2}\times1c=25c$.

Put $d=3$. Take $a=3$ and 6; then $b=24$ and 27, $c=45$ and 48, and $y=\frac{1}{7}x$. Put $x=7$, then $y=1$, and each of the women received 50c or 30c.

NOTE.—A special case of Mr. Gruber's solution is to let y=the price they received for the eggs per dozen and x=the price they received for the remaining eggs. Then $10x$=amount the first woman received, $2y+6x$=amount the second received, and $4y+2x$=amount the third received. Since they all received the same amount, we have $10x=2y+6x=4y+2x$. Therefore $y=2x$. Hence, if they sell them at 1, 2, 3, or 4c each, and 2, 4, 6, or 8c per dozen, they will receive the same sum.

Mr. Charles C. Cross and Mr. M. E. Graber solved the problem by assuming that they sell 7 eggs for a cent and the remaining eggs at 3 cents each. Thus each woman would get 10 cents.

Professor Zerr assumes that the first woman sells 1 egg for 1 cent and the remaining 9 at 6 cents each, receiving, therefore, 55 cents; the second sells 25 eggs for 1 cent each and 5 eggs at 6 cents each; and the third 49 eggs at 1 cent each and 1 egg for 6 cents. This way each would receive 55 cents. ED. F.

ALGEBRA.

161. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

If n quantities are made up of q sets of r each, find the number of permutations s at a time. It is supposed that the quantities in each set are alike, but different from those in the other sets.